NEW BOOK!

Extreme value methods with applications to finance

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The book covers a wide range of topics in modern Extreme Value Theory, Non-parametric Statistics and Financial Risk Management.

It is supplied with a large number of open research problems and over 200 exercises.

Contents

Introduction

Chapter 1. Methods of Extreme Value Theory

1.1 Order statistics

1.2 "Blocks" and "runs" approaches

1.3 Method of recurrent inequalities

This chapter overviews the methods of Extreme Value Theory, including results on upper order statistics, the "blocks approach" and the method of recurrent inequalities.

Chapter 2. Maximum of partial sums

2.1 Erdős-Rényi maximum of partial sums

- 2.2 Basic inequalities
- 2.3 Limit theorems for MPS

Maximum of partial sums is a statistic that combines properties of sums and maxima of random variables. In this chapter we study the distribution of the maximum of partial sums.

Chapter 3. Extremes in samples of random size

3.1 Maximum of a random number of random variables

3.2 Number of exceedances in a sample of random size

3.3 Length of the longest head run

3.4 Long match patterns

One has sometimes to deal with situations where the number of observations is random. For instance, the length of the longest head run (LLHR) in a sequence of random tails and heads is a maximum of a random number of random variables.

LLHR has interesting applications ranging from reliability theory to psychology and finance. Related is the problem of the length of the longest match pattern, which has applications in biology. The chapter presents results on the asymptotic distribution of the maximum of a random number of random variables and the number of exceedances in samples of random size.

Chapter 4. Poisson approximation

- 4.1 Total variation distance
- 4.2 Method of a common probability space
- 4.3 The Stein method
- 4.4 Beyond Bernoulli
- 4.5 The magic factor

The distribution of the number of exceedances can be well approximated by the Poisson law if observations are independent as well as in the case where dependence does not cause clustering of extremes.

This chapter presents two popular methods of Poisson approximation to the number of exceedances: the method of a common probability space and the Stein method.

Chapter 5. Compound Poisson approximation

5.1 Limit theory

5.2 Accuracy of compound Poisson approximation

In this chapter we study the distribution of the number N_n of exceedances over a high threshold. If the observations are independent, then N_n has Binomial distribution, which can be approximated by the Poisson law.

Dependence can cause clustering of extremes, and pure Poisson approximation for N_n may no longer hold. We show that under natural assumptions compound Poisson is the only possible limit law for N_n .

Chapter 6. Exceedances of several levels

6.1 CP limit theory

6.2 General case

6.3 Accuracy of approximation

This chapter deals with the joint limiting distribution of numbers of exceedances of several levels. We present multilevel generalisations of the results of the previous chapter. We evaluates also the accuracy of multivariate compound Poisson approximation.

Chapter 7. Process of exceedances

7.1 One-level EPPE

7.2 Excess process

7.3 Complete convergence to CP processes

Empirical point process of exceedances (EPPEs) are fundamental to the Extreme Value Theory: any result on the distribution of extremes will follow as a consequence if a corresponding theorem for an EPPE is established. This chapter deals with the asymptotic properties of EPPEs.

Chapter 8. Beyond compound Poisson

8.1 Excess process

8.2 Complete convergence

The results of the previous chapter concerning compound Poisson approximation to empirical point processes of exceedances (EPPE) play the role of a central limit theorem. However, the class P^* of possible limit laws for EPPEs is wider than the class of compound Poisson processes. This chapter presents necessary & sufficient conditions for the complete convergence of an EPPE to a given element of class P^* .

Chapter 9. Inference on heavy tails

9.1 Heavy-tailed distributions

- 9.2 Estimation Methods
- 9.3 Tail index estimation from dependent data
- 9.4 Estimation of extreme quantiles

9.5 Estimation of the tail probability

The fact that financial/insurance data often exhibits heavy tails is nowadays the subject of textbooks. In this chapter we present the theory of statistical inference on heavy tails from dependent data.

Chapter 10. Value-at-Risk

10.1 Value-at-Risk and Expected Shortfall

10.2 Traditional methods of VaR estimation

10.3 VaR and ES estimation from heavy-tailed data

10.4 VaR over different time horizons

10.5 Technical Analysis of financial data

This chapter discusses modern approaches to financial risk measurement. We overview advantages and disadvantages of Value-at-Risk and Expected Shortfall as well as the Technical Analysis approach to dynamic risk measurement. 13 exercises.

Chapter 11. Extremal index

- 11.1 Preliminaries
- 11.2 Estimation of the extremal index

In this chapter we discuss the notion of the extremal index in relation to the distribution of extremes.

Chapter 12. Normal approximation

- 12.1 Accuracy of normal approximation
- 12.2 Stein's method
- 12.3 Self-normalised sums of random variables

Many estimators employed in Statistics of Extremes appear self-normalized sums of random variables. This chapter presents results on the accuracy of normal approximation to the distribution of a self-normalized sum of random variables. The results can be applied in order to establish consistency and asymptotic normality of estimators, construct sub-asymptotic confidence intervals, choose between estimators.

Chapter 13. Lower bounds

- 13.1 Preliminary results
- 13.2 Fréchét-Rao-Cramér inequality
- 13.3 Information index
- 13.4 Continuity moduli
- 13.5 Tail index and extreme quantiles

It is widely observed that in many non-parametric estimation problems the accuracy of estimation is worse than in a regular parametric case, estimators are biased, depend on extra "tuning" parameters, and the weak convergence of normalised estimators to the limiting distribution is not uniform.

In this chapter we discuss common features of non-parametric statistical inference. We derive lower bounds to the accuracy of estimation, revealing the interplay between the accuracy of estimation and the "richness" of the class of distributions.

Chapter 14. Appendix

- 14.1 Some probability distributions
- 14.2 Properties of distributions
- 14.3 Probabilistic identities and inequalities
- 14.4 Distances
- 14.5 Large deviations
- 14.6 Elements of renewal theory
- 14.7 Dependence
- 14.8 Point processes
- 14.9 Slowly varying functions
- 14.10 Useful identities and inequalities

This chapter presents many auxiliary results including a probabilistic version of Taylor's formula, useful identities and inequalities, facts concerning dependence, methods of defining random variables on a common probability space, the interplay between distances and probabilities of large deviations, etc.

Index Abbreviations Bibliography