JEFF EVANS

AFFECT AND EMOTION IN MATHEMATICAL THINKING AND LEARNING

The Turn to the Social: Sociocultural Approaches

INTRODUCTION: RECENT DEVELOPMENTS IN RESEARCH ON AFFECT

Twenty to thirty years ago, mathematical affect was considered to comprise basically mathematics anxiety and attitudes to mathematics, typically measured by self-report, paper and pencil measures such as those devised by Richardson and Suinn (1972), and Fennema and Sherman (1976). The former researchers were interested in 'catch-up' programmes for adults intending to return to higher education, and the latter were at the forefront of efforts to increase females' participation in, and achievement in, mathematics courses, especially more advanced ones: in both cases, 'negative' affect towards mathematics was seen as a barrier to their educational goals.[1] In explanations of differences in mathematical outcomes, such as school performance and take-up of mathematics courses, affective variables were considered both to have their own direct effects, and to mediate the effects of social influences (e.g. gender or age) on mathematical outcomes (e.g. Fennema, 1989).

Though it was productive, this early research on affect was constrained by a number of limitations:

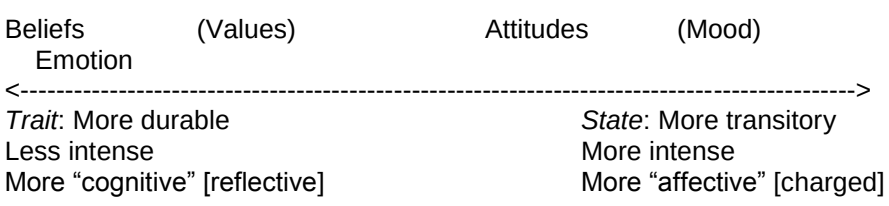
- Affect was measured usually by self-report procedures, which limited what could be tapped to what the respondent was conscious of, and also willing to reveal to the researcher.
- Affect was conceived as trait (enduring) measures, rather than state (transitory) measures, thus limiting researchers' ability to trace the dynamics of problem solving, for example.
- Affective responses were conceived as individual characteristics, thus limiting their capacity to be 'situated' in the social (e.g. classroom, family) context.
- Measures were focussed on a limited range of feelings: anxiety, confidence, liking, enjoyment, perceived difficulty, perceived usefulness.

J. Maasz, W. Schloeglmann (Eds.), New Mathematics Education Research and Practice, 233-255.

- There was a tendency to see affect as “negative” / debilitating of performance, rather than positive / facilitative (or as anything ambivalent or “in between”).

Since the late 1980s, there have been a number of key developments, some related to broadening the scope of mathematical affect, some related to introducing expanded and/or innovative theoretical frameworks.

In a series of important contributions, McLeod (1989a, 1989b, 1992; McLeod & Adams, 1989) argued for the importance of *emotions*, in addition to attitudes and beliefs, which had been the focus in most previous research in mathematics education. He drew on the work of the psychologist Mandler (1989), to present a 'cognitive-constructivist' model for *transitory* emotions, experienced during the *process* of problem-solving, rather than being restricted to measures of *durable* attitudes and beliefs. DeBellis & Goldin (1997) suggested the addition of *values* to the scheme, which might arguably be placed on the durable, less intense side of the spectrum (though they preferred to place the four categories in unordered form as points of a tetrahedron). Evans (2002) suggested the inclusion of *mood*, on the volatile, more intense side of the spectrum.



Sources: McLeod (1992); DeBellis & Goldin (1997); Evans (2002)

Figure 1. McLeod’s Types of Affect

At the same time, there has been continuing interest in mathematics-related *beliefs* both of teachers and of students; see e.g. Leder, Pehkonen & Törner eds. (2002). And although the concept of *attitude* continues to be used in research, it has been subjected to much critical scrutiny (see e.g. Hannula, this volume; DiMartino & Zan, 2001).

The theoretical basis of most research on affect in mathematics education in this period has been mainstream psychology, or (less prominently) mainstream education. There has been only intermittent attention to psychoanalytic approaches; see e.g. Nimier (1977, 1978); Walkerdine (1988); Tahta (1993); Evans (2000); Cabral (2004). All three of these areas share an interest in *motivation*, which, after years of relative inattention in

mathematics education research, is beginning to receive attention by some researchers. This work is summarised by Hannula (this volume); see also Hannula (2006), Mendick (2002), Evans & Wedege (2004).

There has been renewed interest in the biological bases of emotion, in the light of the neuroscientific work of Damasio (1996) and others, which has provided stimulating accounts of the crucial role of emotion in rational procedures, such as decision-making. In mathematics education, see Schlöglmann (2002) and Brown & Reid (2006), who use the concept of *somatic markers*, proposed by Damasio (1996).

There has been some interest in sociological approaches to affect within mathematics education (e.g. Gates, 2006; Evans, Morgan & Tsatsaroni, 2006). But there has been rather more work on developing a somewhat broader grouping of 'sociocultural' approaches. Both of these challenge the psychological emphasis on affect understood as individual characteristics, and emphasise the social basis and *social organisation* of affect.

The key trend in mathematics education research, to which most of the developments described above contribute, is an increasing emphasis on emotions, as compared with earlier periods, when beliefs and attitudes were the main focus. There are a number of reasons for this:

- Mathematics education researchers have tended to take their conceptual frameworks on affect from psychologists (see above), who in turn have tended to focus on 'hotter', more visceral emotion, rather than the whole range of affect in Fig. 1 above (Evans, 2000, pp43-44).
- A focus on emotions allows description of any affect-laden activity, such as mathematical problem-solving, as a *process* which unfolds *dynamically*.
- The activity can be described *in context*, so as to assess the role played by social interaction, classroom culture and pedagogic (and other) discourses in mathematical thinking and performance.
- The more durable forms of affect, attitudes and beliefs can be understood to have a basis in the more transitory emotions (see e.g. Evans, Morgan & Tsatsaroni, 2006).

In the next part of the chapter, I will expand on this emphasis on the emotions, and on how it relates to emphasising the social basis and social organisation of affect in sociocultural approaches. I then go on to describe and to contrast three currently distinguishable types of approach to the study of emotion in mathematics education, all of which can be classified as sociocultural educational research.

EMOTION AS SOCIALLY ORGANISED

A range of social theorists, including the psychologist William James (1890/1950), have considered the different aspects of emotion, and their primacy. When we compare recent analyses from several different disciplines, we find broad agreement on the importance of three aspects of emotional states:
(a) bodily processes, including the brain, but also nerves and organs (e.g. heart, stomach);
(b) behavioural (including verbal) expression; and
(c) subjective experience or "feeling".
We can thus find broadly similar analyses in work by psychologists like Zajonc (1984), Kitayama & Markus (1994), and Buck (1997); by sociologists like Burkitt (1997); by neuroscientists like Damasio (1996)[2]; and by the psychoanalyst Freud (1916-17/1974) on anxiety (see Evans, 2000, pp112-3).

Now, it may seem that all three of these aspects of emotion are individually based – after all, introspection would suggest that they are individual 'experiences'. However, reflection clarifies that certainly (b) methods of self-expression and (c) ways of feeling are at least partly learned in social settings by human beings. Furthermore, social theorists who have argued that the body is the raw material of social and cultural organisation would suggest that (a) bodily processes should be included as social, too (e.g. Grosz, 1994).

This means that there is a wealth of evidence, anecdotal and systematic, that differences, both in modes of behavioural (including verbal) expression and in subjective experience or feeling, are different in different cultures and different social groups (see e.g. Wierzbicka, 1994). Thus it is reasonable to conclude that emotional expression and experience are embedded in social contexts, and thus can be seen as *socially organised* (by the prevailing beliefs and norms, etc.) − just like thinking, learning, or working with mathematics.

So far, we can agree with Markku Hannula (Hannula, this volume) that emotions, emotional states, emotional experience satisfy both of the following:
- they involve physiological reaction, and
- they are functional in human adaptation and social coping.

But we need to question claims made, for example by Markku, that all approaches to the study of emotions need to see them primarily in connection with "personal goals" – *if* the latter are understood in the usual sense as conscious, individually formulated, and/or rationally articulated.

For one thing, emotions may be *unconscious* in the psychoanalytic sense[3] of being pushed into the unconscious, via the operation of *repression*, one of the defence mechanisms. In psychoanalytic approaches, ideas which have strong negative charges, such as anxiety, or which mobilise intrapsychic conflict, have a tendency to meet defences, and thus to be repressed. Therefore, much thought and activity takes place outside of conscious awareness: everyday life is mediated by unconscious images, thoughts and fantasies (Hunt, 1989). This unconscious material is linked to complex webs of meaning (Evans, 2000, Chs.7-10).

In particular, emotions must be understood in connection with *desires* and *fantasies*. There are aspects of these features of human experience that may differentiate them from the characteristics of personal goals, as usually understood. Many desires are unconscious, since they may be felt to be 'unacceptable' or in conflict with the person's desired social image; fantasies are specifically 'unrealistic' or 'irrational' images and narratives that express the desire for some object on the part of the person entertaining them. Both have 'social' aspects, in that desires are connected with social imagery, for example advertising and films, and fantasies can manifestly be shared at the group, professional, or national cultural level (Walkerdine, 1988, Chs. 9 and 10).

1st Interlude: an illustration from Enigma *(2000)*

Both this and the next Interlude present and interpret excerpts from recent films, which portray mathematicians, plus their thinking, and/or their work. This allows me to illustrate the effectivity of films in the way that they articulate powerful elements of social imagery, here discourses about mathematics / mathematicians. In the first excerpt, the themes of desire and fantasy are illustrated in the story of the code-breaking headquarters at Bletchley Park in Britain in World War 2 (Evans, 2003). In this scene, the hero, a mathematician, goes to the home of a woman with whom he had earlier fallen in love. He does not find her there, but he cannot resist entering her room, and recollecting her image, as he smells her perfumes, and, in particular, one earlier meeting with her:

Theme song in the background, they are sitting on a sofa.
She: Why are you a mathematician? Do you like sums?
He, holding a rose: Because I like numbers – because, with numbers, truth and beauty are the same thing … you know you're getting somewhere, when the equations start looking … beautiful. *(He looks at her slightly appraisingly / appreciatively.)*
Then you know the numbers are taking you closer to the secret of how things are. A rose is just plain text…

He hands her the rose; she takes it, but, as he passes it over, a thorn pierces his thumb and makes it bleed. She kisses his thumb; they embrace.

In this scene, the beauty of mathematics is intertwined with that of the rose and that of the woman. He exhibits his desire for these beautiful 'objects', and further, in aligning beauty with truth in mathematics, he suggests a 'higher' form of beauty. His desire to follow "the numbers […] closer to the secret of how things are" suggests a heroic goal shared by many mathematicians, and also attractive to some young mathematics students at school. Others have considered the extent to which this version of 'Reason's dream' can be usefully understood as *fantasy* (e.g. Walkerdine, 1988).

Ways that emotions arise

In the illustration above, the beginning of the scene can be interpreted to show that the male mathematician is experiencing pleasure through entering the room, and smelling the perfume of the woman he loved as these are associated with her. He is also experiencing pleasure through remembering the encounter with her. These re-experienced pleasures derive from the original experience with her, which was imbued with feeling – but they also reformulate that experience, as they reverberate with pleasures experienced in practising mathematics.

In a similar way, many school children and adults would cite experiences in learning and doing mathematics in the classroom, as formative of their 'individual feelings' about mathematics. However we should note the influences at *micro*, *meso* and *macro* levels of the social (see Cobb / Conclusions, this volume). At the meso level, we have the 'socio-mathematical norms' established in the classroom (Cobb et al., 1989), or the form of pedagogic practices (e.g. visible vs. invisible), in which the child is involved (Bernstein, 2000); the latter also have a macro aspect, in that they may be developed and promoted nationally, or even globally, by government policy or other interests. At the micro (face-to-face interactional) level, we can point to examples of experiences in doing mathematics homework within the family; see for example, the case of 'Peter', an undergraduate economics student, who had been constantly 'helped' while at school to do his mathematics homework by his father, and on occasion by one or more of his four 'mathematician' brothers (Evans, 2000).

All of these instances of emotion are experienced by individuals who already have beliefs and attitudes that are to a great extent *culturally transmitted*. This may involve the attention to, and adoption of, views of 'significant others' (Scribner and Cole, 1973). (Thus, three of Fennema &

Sherman's (1976) Attitude to Mathematics scales concerned the student's (perception of) her Mother's, Father's, and Teacher's attitudes towards herself as a learner of mathematics.) But there is also a role for the media and other means of communication, which transmit images of mathematics and mathematicians in popular culture (Appelbaum, 1995; Evans, 2003, 2004; Mendick, 2006)[4].

Emotion can also arise through an association with objects or ideas different from those to which it was originally linked. Psychoanalytic approaches see this as happening through the capacity of an affective charge to move from one idea to another along a chain of associations by *displacement*. A number of examples are given by Nimier (1977, 1978; see Evans, 2000, pp116-9). The following excerpt from another film featuring a mathematician also illustrates this displacement.

2nd Interlude: an illustration from Smilla's Feeling for Snow *(Bille August, Germany / Denmark / Sweden, 1997)*

Here the heroine, who investigates the mysterious death of a young boy in a block of flats in Copenhagen, is also a mathematician. In one scene, where she is having a meal with a man who clearly has strong feelings for her (apparently unreciprocated), she is describing how difficult it was for her to be relocated from Greenland to Denmark, as a young girl:

He: And you were never happy here?
She: The only thing that makes me truly happy is mathematics … snow … ice … numbers [*She smiles*.] To me the number system is like human life. First you have the natural numbers, the ones that are whole and positive, like the numbers of a small child. But human consciousness expands and the child discovers longing. Do you know the mathematical expression for longing? [*He shakes his head.*] Negative numbers, the formalisation of the feeling that you're missing something. Then the child discovers the in-between spaces, between stones, between people, between numbers – and that produces fractions. But, it's, it's like a kind of madness, because it doesn't even stop there…. There are numbers that we can't even begin to comprehend. Mathematics is a vast open landscape: you head towards the horizon, it's always receding … like Greenland. And that's what I can't live without, that's why I can't be locked up….
He: Smilla, can I kiss you? [*She moves away.*]

This scene again associates mathematics with beauty: here we have a beautiful female mathematician herself talking about mathematics. As we listen to her talk, what comes across most strongly is her longing … for numbers, mathematics, Greenland, and the sense of loss as she sees them

"always receding". The original (in this excerpt) feeling of loss and longing appears to relate to Greenland, which itself may stand for another object, such as her dead mother; that feeling is *displaced* onto mathematics, and in turn onto the negative numbers – that part of mathematics which for her "formalises" the feeling of loss, and which she contrasts with the "whole and positive" natural numbers of the young child.

Thus we see that films, and other objects of popular culture are sites for the articulation of discourses within which meanings are defined, images are built up, and hence power is invested. This illustrates another way in which emotions are socially organised.

THE SOCIAL ORGANISATION OF AFFECT: SOCIOCULTURAL APPROACHES

I have argued above that emotion and affect are socially organised, and that a broadly sociocultural approach is needed for investigations in this area. My understanding of developments in this area in mathematics education research at this time is that there are several approaches which certainly have different starting points, but whose trajectories suggest the possibility of developing significant common ground. For the purposes of illustration, I group these sociocultural approaches provisionally under three headings:

- socio-constructivism (SC), based on efforts aiming to bring out a social or 'situated' aspect to work based on the ideas of Piaget and others labelled as "constructivists" (Ernest, 1991; Cobb and Bowers, 1999); see, for example, Op' t Eynde & De Corte (2003), Gomez-Chacon (2000).
- cultural-historical activity theory (CHAT), based on the work of "Soviet psychologists", as developed in the USA and Western Europe over the last fifty years (Leont'ev, 1978); see also Cobb (this volume), van Oers (this volume).
- a discursive practice (DP) approach, which draws on Critical Discourse Analysis (Fairclough, 2003) in socio-linguistics, work on pedagogic discourses in the sociology of education (Bernstein, 2000), and Valerie Walkerdine's poststructuralist analyses, drawing on psychoanalytic concepts (e.g. 1988, 1997); see also Evans (2000), Mendick (2006).

In the following sections, I aim to illustrate the scope of three recent strands of sociocultural work on affect, to highlight theoretical similarities (and to note differences) among these approaches, and to give instances of innovative methodologies. Given space limits, I compare them by referring mainly (but not exclusively) to one 'exemplar' from each. The illustrative reports are:

SC: Op ‘t Eynde, de Corte & Verschaffel, 'Accepting emotional complexity: a socio-constructivist perspective on the role of emotions in the mathematics classroom' (2006)
CHAT: Roth, ‘Motive, Emotion and Identity at Work: a Contribution to Third-Generation Cultural Historical Activity Theory’ (2006)
DP: Evans, Morgan & Tsatsaroni, ‘Discursive Positioning and Emotion in School Mathematics Practices’ (2006).[5]

In order to compare the three approaches to emotion, I use an approach to the analysis of research reports developed by Evans and Zan (2006) [6], drawing on Schoenfeld (2002), and especially Lerman et al. (2002)[7]. Evans and Zan aimed to produce a set of questions and categories that could be used to systematically read and categorise a set of articles from a reasonably homogeneous area, so as to assess commonalities and divergences among different contributions, and the scope of work in the area; see Figure 2.

1. *Conceptual framework*:
(a) What are the *key concepts* and the *basics of the approach* used in researching affect?
(b) How is *emotion* characterised?
2. *Problems addressed*:
(c) What are the *aims* motivating the research at this stage?
3. *Methodology*:
(d) What are the *preferred research methods* for the approach?
(e) What are the *key phases* in the main study reported, and the *research design* (the population of interest, etc.)?
4. *Outcomes*:
(f) What *uses* of the approach are apparent in the main study reported?
(g) What *findings* illustrate the range of the approach, both those which are distinctive to the approach, and those in line with other approaches here classed as sociocultural?

Source: Evans & Zan (2006)

Figure 2. Questions for the Systematic Reading of Research Reports

The categories available for responses to each question, and the way they are applied, are illustrated in the following analyses of each approach.

SOCIO-CONSTRUCTIVIST (SC) APPROACHES

Research Report: Op ’t Eynde, DeCorte and Verschaffel (2006); different features of the same main study are reported in Op 't Eynde et al. (2001) and Op 't Eynde & De Corte (2003).

Key concepts and *basics of the approach*: participation; context; situated; appraisal; component systems; beliefs; motivation. Students' learning is perceived as a form of *engagement* (cf. Wenger, 1998) that enables them to realise their identity through *participation* in activities *situated* in a specific *context*. A student’s appraisal processes, part of his/her cognitive system (itself one of five mutually influencing *component systems* – see below), are in turn influenced by the student’s mathematics-related *beliefs*.

For Op ‘t Eynde et al., *emotions* are conceptualised as consisting of coordinated feedback from multiple processes, which mutually regulate each other over time in a particular context. These processes are characteristic of five different systems (Scherer, 2000):
- the cognitive system, including appraisal processes
- the autonomic nervous system, the basis for arousal
- the monitor system, the basis for feeling
- the motor system, the basis for (behavioural) expression
- the motivational system, the basis for action tendencies.

Emotions are seen as social in nature and situated in a specific socio-historical context, because of the social nature of an individual's knowledge and beliefs – which play a role in appraisal processes, themselves context-specific and fluid.

Aims of the study: Analysing the relation between students' mathematics-related beliefs, their emotions, and their problem-solving behaviour in the mathematics classroom. Given the close relation between emotions and beliefs, investigation of students' emotions can enhance understanding of their beliefs and therefore behaviour.

Methodology: The basics of the approach imply that *mathematical activity should be studied in context*, and that researchers should *take an actor’s perspective* that allows the meaning structure underlying students’ behaviours and emotions to become explicit.

Preferred research methods: Following these methodological implications, the main study adopted a multiple approach to collecting data *within* selected classrooms, involving protocols and video tapes of problem solving episodes, questionnaires, interviews.

The *research design* for the study involved selecting four different classrooms within four different schools in the second year of junior high school (aged 14) in Belgium; the four classrooms represented four different

secondary education tracks, ranging from classical to vocational “levels” (Op ‘t Eynde & de Corte, 2003).

There are two *key phases* in the data collection here. First, a beliefs assessment: all students in the four classrooms were presented with the Mathematics-Related Beliefs Questionnaire (MRBQ). Second, problem-solving behaviour and interviews: four students in each class, selected to represent different ‘belief profiles’, were asked to solve (each of four) mathematical problems, and the process was documented, using a series of records:

- On-line Motivation Questionnaire (OMQ), after the students had skimmed each problem, before actually starting work
- videotaped “thinking aloud” during problem solving
- immediately after finishing, an interview procedure using a Video-Based Stimulated Recall interview (VBSR).

The analysis of the data itself can be divided into four *key phases*. First, the beliefs questionnaires were analysed at the level of the entire sample, using e.g. factor analysis to explore the dimensionality of responses; this provided a basis for each student’s responses to be compared with others’ and categorised into ‘belief profiles’ (‘negative’, ‘mildly positive’, ‘positive’ or ‘highly positive’). Next, for each student, the ‘narrative’ describing the process and experiences of each problem-solving episode for each student was produced using the different data sources; these narratives were content analysed. Third, the relations between students' mathematics-related beliefs, their task-specific perceptions and their problem-solving behaviour could be analysed systematically, with a view to producing explanations. Finally, cross-sectional analysis used the results for the sample of student-episodes as a whole.

Outcomes: The research highlights methodological implications of the theoretical framework presented, e.g. the need to study learning and problem solving in the classroom, and to take account of the different component systems constituting an emotion. This methodological approach is applied and illustrated with the data set.

Illustrative findings: The principal component analysis (of 57 beliefs items) produced four components:
- beliefs about the classroom context, specifically the role and functioning of the student’s own teacher
- beliefs about the value of mathematics, and the student’s competence in it
- beliefs about mathematics as a dynamic and social activity

– beliefs about mathematics as a domain of excellence. (Op ‘t Eynde & De Corte, 2003).

The narratives provided the basis for ‘emotional profiles’, plotting over time the changes in emotions (on a positive / negative scale) experienced (or reported) by each student during each episode[8]. The profile for ‘Frank’ (not his real name) shows a characteristic ‘roller-coaster’ pattern of alternation of ‘positive’ emotions like confidence, happiness and relief with ‘negative’ emotions such as worry, frustration and anger (Op ‘t Eynde & Hannula, 2006, Fig. 2).

Finally, the cross-sectional analysis (n = 16 students) suggested higher anxiety levels among students from the vocational or traditional humanities classes, compared with those from the humanities reform and traditional classical classes (higher social status). Students with a negative or only mildly positive belief profile were more likely to be classified by the researchers as holding avoidance goals towards the problems than those with more positive belief profiles (who tended to hold ‘instructional’, task accomplishment, or knowledge building goals) (Op ‘t Eynde & De Corte, 2003).

To sum up, Op ‘t Eynde et al. deploy a formidable array of methods to analyse the relation between students' mathematics-related beliefs, their emotions, and their problem-solving behaviour within the mathematics classroom. (Their methodological principles (see above) would certainly seem to raise questions about the appropriateness of studying emotions through out-of-class interviews.) However, the range of measurement instruments used, some fairly ‘technological’ and obtrusive (e.g. the On-line Motivation Questionnaire) may have limits if one wishes to preserve the ecological validity of the methodology. This concern would gather force, if the researchers were to follow their aim to deploy “a wider variety of instruments” to study component systems concerned with processes (“e.g. hormonal and physiological”) in addition to the cognitive, as studied here.

Despite its promise to help in focussing on a wider range of possible measurements, the component systems framework does not yet seem to sit comfortably with the emphasis on the social. While it is plausible to argue that the student’s belief structures will provide a ‘social’ influence on the cognitive (including appraisal) system, or the motivational system (though that has still to be explicated in this work), it is less clear how these beliefs will influence the motor, monitor, or autonomic nervous systems. Overall, the view of the social in this socio-constructivist approach, referring to beliefs and the social contexts of the data, does not analyse sufficiently the social structures relevant to the settings in which the students find themselves, for example the hierarchy of types of secondary schooling,

which relates crucially to differences in expectation and resourcing, or indeed the social class backgrounds of the students themselves. This is important, since the interpretation of 'realistic' problems in classroom settings has been shown to depend on the pupils' social background and educational experiences (Cooper & Dunne, 2000).

Socio-constructivist approaches in general are reflected on elsewhere in this book (van Oers, this volume).

CULTURAL - HISTORICAL ACTIVITY THEORY (CHAT)

Research Report: Roth (2006); different features of the same main study are reported in Roth (2003, 2004, in press) and Lee & Roth (2005). Cultural - Historical Activity Theory is discussed elsewhere in this book (van Oers, this volume, and, for an overview, Cobb / Introduction, this volume); see also Engeström (e.g. 2001).

Key concepts: socially organised activity, action, operations, tools, motivation, identity. The *context* for any action is the *activity* in which the subject is engaged; the basic elements of activity include *subject*, *object*, *tools*, *community*, *rules* and *division of labour*. Activities are oriented toward collective motives, which have arisen in the course of cultural historical development; they are organised in the triplet of *activity* / *action* / *operation*: "activities and actions presuppose each other: activities are realized through concrete practical actions, but [conscious, goal-directed] actions are oriented toward the activities" (Roth, 2006). On the other hand, "actions and operations also presuppose each other, as a particular practical action is concretely realized by operations, which are only operated to bring about the action. Among the conditions shaping an operation are the current state of the action and the neurological, biochemical, neuromuscular, and emotional states of the body" (ibid.).

Emotions in this approach come from the body, as described by Damasio (1996), whose findings on the integral role of emotions in decision making are drawn on. Emotion is seen as 'integral to practical action' in two ways: first, 'the general emotional state of a person shapes practical reasoning and practical actions'; second, practical action is generally directed toward "increases in emotional valence" (Roth, 2006, 2004). Here, rather than simply equating to higher levels of pleasure rather than pain, an 'increase in emotional valence' is associated with an increase in 'room for manoeuvre' (a greater choice of actions to choose from) or to being 'better off in the long run'. Emotion is seen as a crucial basis for *motivation* and identity, which derive from it. Motivation is constituted by an "expansion of action

possibilities, [which is] loaded [i.e. associated] with an increase in emotional valence in the context of predictable effort, cost and risk" (Roth, in press; see also Turner, 2002). *Identity* is related to an individual's participation in collective activity, and to the 'recognition' received as a member of the community; this relates to individual and collective emotional valences arising from face-to-face interaction with others.

Aims: This paper aims to extend the relatively cognitive approach of '3rd generation CHAT' to encompass emotion, motivation and identity – and to provide evidence of the need for that. This is to provide the basis for a fuller explanation of performance, notably mathematical thinking and modelling, at work (Roth, in press). Thus this work focuses on the mathematical thinking of adults in the workplace, unlike the other two research projects reviewed.

Methodology: The first *key phase* of this study was Roth's full-scale (four-year) ethnography of a salmon fish hatchery in British Columbia. The *preferred research methods* are thus ethnographic (participant observation), as in much work done in the CHAT approach (e.g. Roth, 2005). When the author decided his claims about emotions required more convincing indicators, this was supplemented by systematic measurement of speech intensity and pitch (Roth, 2006).

Outcomes: The study revisits 3rd generation CHAT theory, and contributes to a significant revision, with the illustration of inclusion of conceptions of emotions, motivation and identity.

Illustrative findings: The ethnographic findings described the emotions of pleasure expressed by one of the fish culturists ("Erica") on finding that the fish under her care had grown to satisfactory sizes; these findings were supported further by the measures of the actor's voice pitch. Her use of a PC and a range of mathematical representations (graphs and statistical summaries) allowed her to better monitor the progress of 'her' fish, thus increasing her 'room for manoeuvre' and her feelings of pleasure (positive valence). These actions also help her to be recognised as a bit of a "geek" ("nerd"), an aspect of her identity in the activity system of the fish hatchery. The researcher sees the examples provided by the long-term ethnographic contact as hinting at a "dialectical relation linking individual and collective emotion" (Roth, 2006): for example, he is able to chart the change in *mood* – a moderately volatile type of affect (see Figure 1 above), here experienced *collectively* – in the fishery when operating costs increased, and government funding declined.

To sum up, this series of papers argues firmly for a view in which cognition and emotion are seen not only as mutually influencing, but also as having

'inner connections in activity' (2006). Emotion is seen as a crucial basis for motivation and identity, which derive from it. Including emotion, motivation and identity in 'Third-Generation CHAT' will certainly enhance the theory's ability to contribute to the understanding of practical action. It also provides the basis for dialogue with the other sociocultural approaches examined here.

DISCURSIVE APPROACHES

Research report: Evans, Morgan & Tsatsaroni (2006); see also Evans (2000, 2002, 2003a), Morgan, Tsatsaroni & Lerman (2002), Morgan (1998).

Key Concepts: discourse, practices, positions, positioning, subjectivity, power. Discursive approaches focus on specific societal / institutional *practices*, which are recurrent forms of behaviour / action. A *discourse* then is the system of ideas / signs organising and regulating the related practices, in a way that crucially connects with social relations of *power*. Discourse has several functions:

- defining how certain things are represented, thought about, and practised
- providing resources for constructing meanings, and accounting for actions
- helping to construct identities and subjectivities, which include affective characteristics and processes (Hall, 1997).

Power is exerted in micro social interactions, in 'meso' institutional contexts, and in the wider culture, including by policy-makers and by the media within popular culture (Appelbaum, 1995).

A key concept is that of *positioning*, a process whereby an individual subject takes up and/or is put into one of the *positions* which are *made available* by the discourse(s) at play in the setting. This is how the approach allows for a mutual influencing of social and individual: the social setting makes available specific practices, and individuals retain a degree of agency, to strive to position themselves in available (or 'created') positions. The social produces other effects: different positions are associated with membership of different social groups (class, gender, ethnicity), and with different degrees of power. In this approach, a person's *identity*, which includes more durable affect such as attitudes and beliefs, comes from repetitions of positionings, and the related emotional experiences, in a context of a personal history of positionings in practices.

Emotion is related to *desire*, which is considered to permeate the workings of language. Thus emotion can be visualised as a charge attached to ideas and the terms in which they are expressed. This charge has a physiological, behavioural (including verbal) expression, and a subjective

'feeling' aspect (see above). This allows emotion to be seen as 'attached' to ideas (cognition), but in ways that are fluid, not fixed. Some of this fluidity can be seen as related to psychic processes of *displacement*, where meanings and feelings flow along a chain of ideas (or signifiers) and *condensation*, where meanings and feelings 'pile up' on a single signifier (Evans, 2000). Thus, the psychic / 'individual' and the linguistic / social interconnect.

Aims: This paper aims to "show that emotions are socially organised phenomena, which are constituted in discourse, shaped in relations of power, and implicated in constructing social identity" (Evans et al., 2006). In theoretical terms, the work also has interdisciplinary aims, to bring together ideas on the form of pedagogic discourse from the sociology of education, analyses of the process of positioning from social semiotics, and insights into the dynamics of the unconscious from psychoanalysis. In practical terms, it aims to sensitise teachers, teacher educators and policy makers to the (often neglected) importance of emotions in the learning (and use) of mathematics.

Methodology: Because of the emphasis on detailed semiotic analysis of texts, methods which produce transcripts of social interaction are appropriate. Thus, *preferred research methods* include classroom observation and also interviews. This study analysed the transcript of interaction in a classroom episode selected by a colleague of the authors as being possibly fruitful for studying emotion[9]. The episode analysed involved three boys, working together on a mathematical task, in an 8th grade classroom in Lisbon, Portugal. Walkerdine (1988) used transcripts of mothers and daughters discussing everyday tasks at home. Other studies have used interviews with teachers or students (Morgan et al., 2002; Evans, 2000) or questionnaires (Evans, 2000).

Two key *phases of analysis* of the transcripts (and videotapes), structural and textual, are based on the interdisciplinary theoretical approach. First the *structural phase* uses Bernstein (2000)'s sociology of education to show how pedagogic discourse(s) *make available* particular positions to individuals. For example, the discourses at play in school invariably include evaluation practices, which make available positions of *evaluator* and *evaluated*. The 'official discourse' (often 'traditional') is contrasted with 'local pedagogy' (in this classroom, relatively 'progressive'), where students may be encouraged to evaluate each other's work. Furthermore, other discourses from 'outside', including those from the peer-group and from the youth culture, are also at play in the classroom. Conflicts between expectations of different practices may elicit emotion.

Second, the *textual phase* has two functions: (a) showing how positions *are actually taken up* by subjects as positionings, in social interaction, and (b) providing material for *indicators* of emotional experience. For these purposes, social interaction is itself represented as text, e.g. via transcripts. Examples of how interpersonal aspects of the text are used to establish particular participants in particular discursive positions include: claims to know or to understand (which are powerful in educational settings), and the use of repetition or hidden agency (passive voice).

Indicators of emotional experience can be divided into (i) those understood within the institutional subculture and/or wider culture, drawing on the everyday 'folk culture' of participants, and (ii) indicators suggested by psychoanalytic insights. Examples of (i) include: verbal expression of feeling; behavioural indicators (facial expression, tone of voice); use of particular metaphors, e.g. a student claiming to be "coasting" in mathematics (Evans, 2000, p214). Examples of (ii) include mainly indicators of *defences* against strong emotions like anxiety, or conflicts between positionings (see above; Evans 2000), e.g. 'Freudian slips', such as a 'surprising' error in problem solving, behaving 'strangely' (e.g. laughing 'nervously'), *denial* (e.g. of anxiety), as in 'protesting too much' about how confident one is.

Outcomes: The authors apply their theory to a 'critical case': this is classroom (rather than interview) data, involving several students interacting, and not originally collected for studying emotion; they argue that the results indicate a wider scope for the study of emotions, using this (and other sociocultural) theory than might originally have been expected.

Illustrative findings: At first sight, there is little evidence of the pupils directly *expressing* emotion, though some anxiety is arguably being *exhibited* (cf. Evans, 2000). Yet in one part of the lesson, when the boys are working on their own, in a small group, according to the norms of the local (relatively 'progressive') pedagogy, indicators of several emotions are observed, e.g. excitement on the part of two boys, as well as suggestions of anxiety on the part of all three; the authors argue that these are associated with each participant's positioning in discursive practices. Moreover, when this episode is compared with a later one when the teacher intervenes with a strong suggestion as to how the problem should be addressed mathematically, the traditional pedagogic relations are re-established, with a consequent reduction in the space available for the expression of emotion. In general, the classroom episode shows the importance of evaluation practices – applied to oneself and to others – in establishing an individual's positionings and identity. For example, the authors show how (more durable, less context-specific) *identities* might be produced from *repeated positionings* in this way: one boy ("Mario") in the small group,

becomes 'identified' as weak in problem solving, as a result of repeated use of criteria of evaluation that are clearly not from the school mathematics – but rather from 'outside', youth culture discourses. Further, the boy's apparent anxiety, which at first seems to be related to the mathematical task, may be better interpreted as being about being included socially in the group. This illustrates the fluidity with which emotion can be displaced from one 'object' to another (cf. Evans, 2000, Ch.10).

The DP approach shows how meanings and emotions are socially organised in pedagogic contexts. The mathematics and the pedagogic discourses (especially evaluation criteria) interact with other discursive resources and personal histories of individual students, enabling certain positions and creating links and contradictions, thereby opening up spaces within which emotion may occur. The dynamics of the interactional practices lead to ways in which the *positions available* in discourse are realised as *positionings* in practice, thereby allowing space for emotions to be experienced, and sometimes expressed.

The cognitive and the affective are treated as intertwined by showing how discourses and positionings shape both. This avoids the assimilation of the affective into the cognitive – since conceptualising emotion as a charge attached to ideas and the terms in which they are expressed allows the researchers to understand emotion as 'attached' to cognition, but in ways that are not fixed, but fluid.

In connection with the low levels of emotional expression apparent in the classroom studied, it is worth noting that Evans (2000) found many more instances of emotion being expressed, in his interviews with social science undergraduates. However, it can be argued that the difference in levels of emotional expression could be set down to the different discursive constitution of the two contexts, that is, the different positions offered by the classroom and the research interview settings (Evans, 2000, Ch.9; Evans, 2003).

CONCLUSIONS [10]

Common ground

1. Taken together, the three approaches considered here, as illustrated by the selected studies, show that a sociocultural programme of research focused on a shared problem, the role of emotion in mathematical thinking, can benefit from each of the approaches. Comparison of the three studies on *aims* reveals similar motives for including emotions in the theoretical

framework, such as the need of a richer understanding of mathematical thinking and behaviour overall, and its relation to social factors.

All three view mathematical thinking as 'hot', as infused with emotion – in contrast with the commensense view of mathematics as 'cold'. In terms of *key concepts*, the socio-constructivists (SC) understand emotions as related to coordinated feedback from mutually regulating multiple processes based in the person's 'component systems', in particular, the appraisal, monitor and autonomic nervous systems, highlighting, in their work so far, the effect of knowledge and beliefs on this appraisal. The cultural - historical activity theoretical (CHAT) account here sees the person's emotions as related both to relatively conscious efforts to maximise 'emotional valence', and also to the non-conscious "states of the living body" (2006); in this way, it is "reciprocally" (dialectically) related to practical reasoning and action. The discursive practice (DP) approach sees emotion as an affective charge which may be attached to ideas (carried by signifiers), and shows how a range of emotions are associated with each subject's positioning in practices, and especially conflicts in positioning.

These accounts no longer see emotions towards mathematics as largely 'negative' or debilitating, as was the case in earlier research programmes, but often show them as 'positive' / facilitating. Indeed the positive / negative categorisation may be problematical (see also Hannula, this volume), and several types of ambivalence have been shown here, e.g. that due to positioning conflicts of the boys in DP; of 'Frank', torn between the imperatives of stopping to think and performing quickly on the 'test', in SC; or of 'Erica', caught between her drive to do her job "at 300%", and her anxieties about money, once she had been laid off from her job.

2. Further, all three approaches stress the importance of the social, the 'context' of learning. The SC conceptualisation aims to capture this via careful measurement of knowledge and especially beliefs, and also through taking account of the type of course and/or school. However, the effects of these contexts and of social types generally, cannot be captured only by commonsense understandings or 'natural' labels (such as 'school maths' or 'workplace maths'). The DP approach shows how to describe a person's positioning within the discursive practices constituting and regulating their context of action. The CHAT report sees activity within a community, with its collective "motives", and located culturally and historically, as the context. Here any idea of social regulation is so far implicit, or simply assumed, and the role of power less fully sketched.

3. Comparisons of *methodology* reveal multi-phase, multi-method procedures, which differ in specific ways (described briefly above) among approaches. A range of methods has been used, including self-completion

questionnaires; systematic physiological measurement; behavioural measurement; several types of interview; participant observation. As is required to deal with dynamic processes, all have methods for capturing the fluidity of emotion: for example, attention both to facial expression and to "thinking aloud" while problem solving in SC; speech intensity and pitch in CHAT; detailed semiotic analysis of verbal interaction transcripts in DP.

Possibilities for developing a programme of research focused on a common problem

4. It is natural to reflect on whether the commonalities among approaches suggest a basis of dialogue and further work, and hence ways of avoiding a proliferation of approaches to the study of emotion in mathematics education. Here, we can only aim to pose several tentative questions.
(a) Are there any overlaps in the key concepts used, that might allow 'fruitful mutual challenges' among approaches? For example, in what essential ways do 'activities' (CHAT) and 'discursive practices' (DP) differ as a context of thinking? Does CHAT have an analogue of 'positioning'? Can the sets of beliefs so carefully investigated in SC shed any light on the content of different pedagogic discourses (DP), or on the structure of the activity system (CHAT)?
(b) The term 'unconscious' is used in three distinct senses here: (i) *routinised*, not needing conscious attention, as with *operations* (in CHAT); (ii) '*autonomic'* as for physiological processes, such as the heartbeat; and (iii) *repressed* via defence mechanisms into the (Freudian) unconscious (in DP). Distinguishing among these more carefully may allow useful conceptual distinctions in the various approaches.
(c) The SC and CHAT approaches in their current versions focus more strongly on motivation; their conceptions could be of value for developments in DP.
(d) At the same time, the SC and the CHAT approaches are at risk of using an approach to the 'social', which relies on only commonsense or 'natural' labels – and which therefore lacks explanatory power. The DP approach can show how the context is constructed through discourses that give it meaning and that serve to locate participants in positions of power, feeling and capability.

5. Psychoanalytic insights appear to be taken up centrally only in the version of discursive approaches presented here. Nevertheless, they pose a challenge to any strongly cognitivist point of view that emphasises thinking as largely 'conscious' and normally bound by rationality. This is because many emotional reactions, and even beliefs, including those relating to mathematics (etc.) are often not conscious, much less rationally arrived at. Thus, the play of desire and fantasies may invest mathematics and mathematical objects with strong emotional meaning. Feelings like

anxiety can be *displaced* to mathematical objects from others, via movement of emotional charge along a *chain of signifiers*: so what seems to be 'mathematics anxiety' may relate to anxiety from other practices. Thus emotion may *transfer* across practices (Evans, 2000), like ideas, perhaps having originated in early relationships, or in images in popular culture (e.g. in films, as illustrated above).

Psychoanalytic insights also suggest that certain beliefs and behaviours are *defensive* (against anxiety and conflict). These insights provide possible explanations for what would otherwise be surprising cognitive 'slips'. These insights may also explain the *transference* of the focus of the student's feelings from parent to teacher.

6. This re-reading of three apparently distinct approaches to broadly 'socio-cultural' research on the role of emotions in mathematical thinking and learning, against an account of developments in this area over the last 25 years, has shown a non-negligible amount of common ground, some possibilities for 'mutual challenging' among the separate approaches, and some promising areas for development in conceptualisation of several key areas, including 'the social'. Thus this discussion opens several areas for further research.

(a) Both 'motivation' and 'identity' have been marked here as of interest in the affective area; the former in particular has been neglected in mathematics education research until recently, but is featured here in the SC and CHAT reports discussed.

(b) Each of these studies offers suggestions as to how to rethink the links between beliefs and attitudes seen as durable aspects of individual 'identity', and transitory emotions.

(c) The SC approach's emphasis on the cognitive appraisal systems, and their relations with beliefs, provides an impetus to bring in the study of other systems; the indicators of voice pitch and intensity used by the CHAT report may help in this respect.

(d) The CHAT study of working adults raises the issue of child vs. adult differences in affective patterns and emotional experience.

(e) The DP approach especially suggests studies of the ways that popular culture has effects on emotions, e.g. using representations of mathematic(ian)s in films.

(f) These three sociocultural approaches together raise questions like:

- When should there be an emphasis on "enjoying maths" in class – and are there any occasions when it should not be emphasised?
- Should educational policy makers try to control emotions in schools, or require teachers to develop students' 'emotional literacy'?
- In a classroom where 'emotional literacy' is emphasised, which different social categories of student (in terms of gender, social class, ethnicity) would stand to gain or to lose?

NOTES

[1] Indeed, Fennema and Sherman (1976) produced a whole battery of questionnaires for attitudes towards mathematics, including Attitude to Success in Math, Confidence in Learning Mathematics, Usefulness of Mathematics, and Math as a Male Domain.

[2] Though, for Damasio (1996), 'emotion' includes only (a) and 'feeling' includes (c)

[3] For two other senses of the term 'unconscious', see the Conclusions.

[4] We might even consider the cultural transmission of images and discourses about emotions in other settings in society (e.g. Hartley, 2004).

[5] Two of the articles, Op 't Eynde et al. and Evans et al., are published in the same journal issue (*Educational Studies in Mathematics: Affect in Mathematics Education: Exploring Theoretical Frameworks, A PME Special Issue,* vol. 63, no.2), and, as part of their analyses, address the same case study, that of 'Frank' (Op 't Eynde & Hannula, 2006).

[6] This section owes much to collaborative work with Rosetta Zan; see Evans & Zan (2006).

[7] Lerman et al. (2002) systematically analysed a sample of research papers published in mathematics education journals and the annual research conference (PME). Their basic idea is that, like any other kind of data, scientific texts need to be systematically read and interpreted. Their approach used a set of questions and categories derived from theoretical resources.

[8] These emotional profiles recall the 'Mood maps' presented by Gomez-Chacon (2000), where, however, the students were coding their own emotions.

[9] The original data set was collected by Madalena Santos for research with a different focus (Santos & Matos, 1998).

[10] These Conclusions have benefited from collaborative work with Rosetta Zan, Anna Tsatsaroni, and Candia Morgan; see Evans & Zan (2006); Evans, Morgan & Tsatsaroni (2006).

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Jeff Evans
Mathematics & Statistics Group
Middlesex University
London UK