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Building Bridges: Reflections on the problem of transfer of learning in mathematics

Jeff Evans

The takeup of opportunities for applying formal learning outside the school is often disappointing - to teachers, parents, employers, and many pupils. Not surprisingly, there is much controversy among researchers in mathematics education and related fields, as to the reasons. Here I argue that neither traditional or constructivist views, with their simplistic faith in the basic continuity of knowledge across contexts, nor currently popular 'insulationist' views such as the strong form of situated cognition, which claims that transfer is basically not possible, are adequate. Instead, I analyse why transfer is problematical in principle, and undependable in practice. I recommend an alternative approach for building bridges between practices, based on analysing the discourses involved as systems of signs, and looking for appropriate points of inter-relation between them. In this reconceptualisation of the 'problem' of transfer, the role of affective factors, previously under-examined in the literature, is highlighted.

Keywords: transfer of learning, adults and mathematics, situated cognition, discourse theory, poststructuralism, affect, emotion

1. Introduction

If schooling is to be relevant to settings and activities outside itself, then we need some account of how learning from the school can be applied in, or 'transferred' to, other contexts. The *transfer* of learning refers in general to the use of ideas and knowledge learned in one context in another. This might take one of several forms:
(i) the use of a school subject like mathematics outside of its own domain, say in physics or economics;
(ii) the application of knowledge from pedagogic contexts to work or everyday activities; or
(iii) the 'harnessing' of out-of-school activities for the learning of school subjects.
Other forms of 'recontextualisation', e.g. the reformulation of academic discourses as school subjects, are related; see below.

Here I am especially interested in issues around (ii), particularly applications in work contexts, and (iii), 'harnessing'. These are clearly crucial issues for schooling, higher education, adult education, and for training. And especially so for mathematics, which is claimed to have wide applicability across the curriculum, and outside the school or college.

2. Divergence of Views on 'Transfer'

Questions around transfer are strongly contested, and a variety of views proliferate in educational circles, as well as in psychology and sociology. The discussion has been especially vibrant in mathematics education, where we are currently offered several conflicting approaches. In this section I present an overview of the views on transfer involved in five approaches: 'traditional' views, constructivism, the 'strong form' of situated cognition, structuralist views, and poststructuralism. Below I consider the Brazilian research programme (Nunes et al., 1993), and, in sections 3 and 5, I develop my view based on insights from poststructuralism, which I argue offers the most promising basis for a viable position.

I begin with *traditional* approaches. These can be seen to include views favouring the use of behavioural learning objectives (e.g. Glenn, 1978), 'basic skills' approaches, and 'utilitarian' views such as those of the Cockcroft Report (1982). They share several important ideas. Teaching and learning are seen as involving the transmission and internalisation of a body of knowledge (and skills). A problem or 'task' (or 'skill'), and the mathematical thinking involved in addressing (or producing) it, are seen as able to be described adequately in abstract terms, with little or no reference to the context - e.g. simply as 'proportional reasoning'. Hence it is claimed to be possible to talk about 'the same mathematical task' occurring across several different contexts. Therefore traditional views expect that the transfer of learning, e.g. from school to everyday situations, should be relatively unproblematical - at least, in principle, for those who have been properly taught.

A number of problems with these views can be summarised. It has been found difficult to describe a task, in ways that are abstracted from the context, so the notion of the 'same mathematical task' in different contexts is highly problematical (Walkerdine, 1988, pp. 51ff.; Newman et al., 1989, ch.3). Where there is an attempt to describe the context of a problem, traditional views generally consider it to be indicated (unproblematically) by its wording, and that context is normally just *named* - as 'school maths', 'consumer maths', etc. Thus, the context is usually described '*naturally*' - rather than being *analysed* for its *socially* constructed qualities (e.g. Atweh et al., 1998).

In addition, the methods of addressing a task have been shown to vary a great deal across different contexts - for example, in terms of the methods of calculation or types of representation (e.g. oral vs. written) used (Nunes et al., 1993; Lave, 1988). And the levels of perfomance of what appears to be 'the same task' have varied dramatically across different contexts (*ibid*.)

Not surprisingly, studies focussed on the problem of transfer, from school to outside activities, suggest that much teaching has disappointing results in this respect (e.g. Boaler, 1998), and students often 'fail' to acomplish it (e.g. Molyneux and Sutherland, 1996). One reason may be that people do not 'spontaneously see' the transfer (the task, the goal) their teachers (or their managers) have in mind. And, even if they see it, they may not be motivated to carry it out (NOTE 1). In such cases, a researcher may conclude that a 'mathematical' signifier is not recognised as such, whereas it may be recognised, but its mathematical meaning be undermined by competing values related to other discourses, (Dowling, 1991), or by affective conflicts (see next section).

Even where a researcher considers a learner to 'transfer' school algorithms in order to address problems from within an everyday practice, the traditional view that it is straightforward is contested. Thus Saxe rejects the traditional view of transfer as an 'immediate generalisation or alignment of prior knowledge to a new functional context', and prefers to conceive of it as 'an extended process of repeated constructions ... of appropriation and specialisation, as children repeatedly address problems that emerge again and again in cultural practices' (1991, p.235; see also Masingila et al., 1996).

Thus, in recent years, several strong critiques of the traditional view have emerged. These can be analysed, using Muller & Taylor's (1995) distinction between two tendencies in curriculum development and change: *insulation* and *hybridity*. They focus on the idea of *boundaries* between e.g. school and everyday knowledges, or discourses, or cultures. For them, insulation stresses

> the impermeable quality of cultural boundaries, of textual classification, of disciplinary autonomy.... Hybridity, by contrast, stresses the essential identity and continuity of forms ... of knowledge, the permeability of classificatory boundaries, and the promiscuity of cultural meanings .... learning to 'cross-over' cultural boundaries is, or should be, the aim of all pedagogy. Questions of judgement and of classificatory integrity take second place to the goal of individual access to learning.
>
> (Muller and Taylor, 1995, p.257)

Thus, an hybridiser tends to believe both (i) that boundaries between contexts *are* low - and (ii) that they *should be* low, lowered, and/or readily crossed. And the two beliefs tend to go together, in the thinking of traditionalists: they tend to see tasks in many contexts as being essentially mathematical, and to believe that the transfer of learning to new contexts should be straightforward.

And so for constructivists. Muller & Taylor (1995) aim to show the unintended consequences of unreflective espousal of 'constructivist' challenges to the traditional view on transfer. For them, writing in post-apartheid South Africa, 'constructivism(s)' include radical constructivism, social constructivism, and particularly ethnomathematics; these have political and pedagogical commitments to challenging a school maths dominated by academic mathematics as being a 'sharply located' kind of knowledge - Eurocentric, imperialistic, dominated by male values. For Muller and Taylor, the constructivists are 'strong hybridisers whose pedagogy assumes a flattening of the everyday / school boundary' (pp.267-8). Somewhat surprisingly perhaps, we can see that the constructivists share the hybridising commitment, and the flattening assumption, with the traditionalists described above.

However, if the constructivists are liberal and egalitarian between different forms of knowledge, traditionalists are rather imperialist in their desire *to privilege mathematics* as a special kind of (abstract) knowledge, and to apply it widely.

In contrast, a third position, which might be called the strong form of situated cognition, bases its position on Jean Lave's *Cognition in Practice* (1988), and argues that there is a *disjunction* between doing maths problems in school, and numerate problems in everyday life. This is because these different contexts are characterised by different *structuring resources* - different ongoing activities, different social relations, different cultural forms of quantity (e.g. money), and so on. Further, people's thinking is *specific* to these disjoint practices, and settings. Thus transfer of learning from school or academic contexts to outside ones is pretty hopeless.

In terms of my classificatory framework, the strong form of situated cognition could be classed as having an insulating commitment, and a belief in boundaries between practices as high - and, against the traditionalists, as not wanting to privilege mathematics as a special kind of knowledge.

The strong form of situated cognition is presented here as an 'ideal type', but it still has many proponents. The work of Jean Lave herself has moved on. Her recent studies (Chaiklin and Lave, 1993; Lave, 1996a) focus on describing learning within *communities of practice* (Lave and Wenger, 1991). This work is no longer so concerned to stress discontinuities between practices: it acknowledges that no practice could ever be completely closed, and that a community must be understood in relation to other tangential, or overlapping, communities. The approach now consists of identifying communities of practice which are interdependent, and studying the bridges between them, particularly the social relations and identities across them (Lave, 1996b).

However, there are a number of problems and gaps in the account given by situated cognition. First, in its strong form, the view threatens a cul-de-sac (Noss & Hoyles, 1996b, ch.2): there is a proliferation of differently situated types of mathematical thinking, with high boundaries between them, and the use of one type of thinking in another context is indeed pretty hopeless. Second, in this approach, there seems to be an assumption that practices and communities of practice can be seen as 'natural' - whereas I argue that *analysis* of the bases of different practices (and communities) is required. Although Lave and some others researching within broadly situated approaches (e.g. Greeno et al., 1993) mention 'sign systems', etc., I emphasise the systematic consideration of the effects of language and discourse underlying practices in different contexts.

For an indication of how this concern with language might help clarify the idea of boundaries between practices, we can turn to certain structuralist approaches. Muller and Taylor draw on Basil Bernstein's (e.g. 1996) sociological discussion of boundaries between knowledges, where different knowledges are seen as different discourses or systems of language. School knowledge is reinterpreted or transformed from academic knowledge in the universities, by processes of *recontextualisatio*n, which produce a new discourse with distinctive principles of selection, ordering and focussing. Thus, Bernstein is an insulator, for whom 'curricular knowledge is part of that large class of esoteric discourses, separated from everyday knowledge by a hard boundary that we weaken at our peril' (Muller and Taylor, pp.262-3).

Paul Dowling (e.g. 1994) develops these ideas by showing that the mathematical texts prescribed for 'lower ability' students and which incorporate numerous examples intending to model every day situations, have the consequence of excluding their readers from the 'esoteric' discourse of school mathematics proper. For Dowling, the recontextualisation of everday life material into the curriculum ends up by being neither 'real maths' nor 'real life'. Besides distorting the everyday setting - in which the 'lower ability' learners are meant to feel 'at home', it also inculcates an anodyne view of mathematics as a series of algorithmic solutions - very different from the view of mathematics as a connected set of generalisable principles, into which only the 'higher ability' students are inducted. This structuralist work points to ways of analysing practices and the boundaries between them, but there is still the threat of arriving at the same sort of cul-de-sac as with the strongly situated approach: 'Dowling's strong position would seem to imply that school mathematics should incorporate no “real world” examples' (Muller & Taylor, p.268).

Valerie Walkerdine's poststructuralist work is sensitive to the need to avoid the pitfalls revealed by Dowling. Like the constructivists, she is committed to bridging the space between everyday and school knowledge, but, unlike them, rather than assuming it away, she sees the importance of *theorising* the boundary. As with situated cognition, she recognises different practices as in principle distinct, but sees this distinction as requiring analysis, rather than leading to despair about bridging.

The discussion so far shows that two issues need to be addressed, so as to formulate the problem of 'transfer' satisfactorily. They are:

(1) how to define and delineate the contexts of thinking, activity and learning, and the related practices at play in them; and

(2) how to describe the relations between practices, e.g. what the boundaries or bridges between them might be like.

A third issue is crucial, as I argue below:

(3) how to acknowledge the importance of affect, motivation and so on, so as to avoid separating thought, feeling and value.

The initial locating of Walkerdine's position above suggests that certain approaches drawing on discourse theory, and poststructuralist insights, can make a contribution to elucidating these issues (e.g. Walkerdine, 1988, 1997; Walkerdine and Girls & Maths Unit, 1989; Taylor, 1989; Muller and Taylor, 1995; Dowling, 1995; Evans and Tsatsaroni, 1994, 1996; Evans, 1999a).

3. Conceptualising Practices, Boundaries and Bridges

3.1 Describing the contexts, and the practices at play

The approach I am advocating focusses on *practices*: examples would be school mathematics, academic (research) mathematics, work practices such as nursing (Pozzi et al., 1998) and banking (Noss & Hoyles, 1996a), apprenticeship e.g. into tailoring (Lave & Wenger, 1991), and everyday practices such as shopping (Lave, 1988). Each context is constituted by one or more practices, and by the related *discourses*. Discourses are systems of ideas expressed in terms of *signs*. These discourses give meaning to the practice by expressing the *goals* and *values* of the practice, and *regulate* it in a systematic way, by setting down standards of performance.

Important practices are associated with a community of practice, a subculture of individuals with (some) shared *goals*, and a set of *social relations* (power, difference) with different members of the community taking up different *subject-positions*. For example, the basic positions available in school mathematics are normally 'teacher' and 'pupil'; in shopping or street-selling, they would be 'seller' and 'buyer'. In a particular setting, we can analyse the practices *at play*, that would be involved in the positioning of participants (NOTE 2).

This approach, like situated cognition, recognises different practices as in principle distinct, as discontinuous - e.g. school maths and everyday practices like street selling. But, using the approach recommended here, we can go further - to analyse the differences. Language and meaning in the discourses involved can be analysed by considering relations of signification - relations of similarity and difference between terms or *signifiers* (words, gestures, sounds, etc.) and *signifieds*, and devices such as metaphor and metonymy. For example, a knowledge of counting will help with playing cards - but only up to a point: to play bridge or whist with the standard deck, you must know the ranking of numbers up to ten - but also that the Ace, though signified by *a single* heart, etc. will beat all others in that suit.

So far this draws on de Saussure's structural linguistics. Going further, various writers have shown how to use poststructuralist ideas about the inevitable tendency of the signifier to slip into other contexts, thereby making links with other discourses, and producing a play of multiple meanings - so as to provide insight into meaning-making in mathematics; see examples below of the different possible meanings of 'more', and of 'shopping with mummy'; also Walkerdine (1988, Ch.2) on children's use of language to indicate relations of size, Brown (1994) and Evans and Tsatsaroni (1994). Thus, rather than attempting to specify the context of a school maths problem by looking only at its wording - or by naming the context as if simply based in 'natural' settings, we can describe it as socially constructed in discourse - through attention to particular signifiers and their relations in texts, such as interview transcripts (see 'Donald's' case study below).

Different practices may also be characterised in terms of their 'well-known results' (cf. Lawler, 1981), and their familiar methods. An example can be given by contrasting a street seller's calculation of the cost of 10 coconuts (@ 35cr. each) - as 105 (three 35's, a 'well-known result'), plus 105, plus 105, plus a final 35 - with a pupil's doing 35 x 10 (Nunes et al., 1993, Ch.2).

3.2 Describing the relations between practices, the boundaries or bridges between them

To build bridges between practices, one must try to identify areas where out-of-school practices might usefully 'overlap' or 'inter-relate' with school mathematics. This requires first of all that distinctions are made between those relations of signification in the learner's everday practices that provide fruitful 'points of articulation' with school maths, and those that may be misleading (Muller and Taylor, 1995), as in the whist example (above). Another example of a misleading inter-relation would be an attempt to harness young children's everyday understanding of 'more' to teach the comparison of quantity at school. The problem is that in home discourses, the opposite of 'more' is *no more* (as in 'no more ice cream for you'), but, in school discourses, 'more' forms an oppositional couple with *less* (Walkerdine and Girls and Maths Unit, 1989, pp.52-53). Here the signifier 'more' signifies differently in home and school practices.

Besides attending to fruitful (non-misleading) points of inter-relation, we must structure the pedagogic discourse so as to work systematically through a process of translation. This involves 'prising apart' signifiers and signifieds linked in one set of signs, and reinserting them a new set of signs.

This is done through the construction of ‘chains of meaning’. A very simple example is that of a mother who uses a discussion with her child on the number of drinks needed for a party of the child's friends to teach the child to count by the following transformations from one step to another (Walkerdine, 1988, p.128ff):

Step				
1	Child (*signified*) Name (*signifier*)			
2		Name (*signified*) Finger (*iconic signifier*)		
3			Finger (*signified*) Spoken Numeral (*symbolic signifier*)	
4				Spoken Numeral (*signified*) Written Numeral (*symbolic signifier*)

At the first step, the mother-teacher, encourages the child to form a sign linking the name of each child *(signifier*) with the 'idea' of that child (*signified*). At each subsequent step, the signifier from the previous stage becomes the new signified, which is in turn linked to a new signifier (gesture, numeral). Here, each different step does not really represent a different discourse, but the overall chain nevertheless shows how a series of carefully constructed links between signifiers and signifieds could provide the bridges for crossing boundaries between discourses - here, the basis for transfer between home practices and school maths. (This scheme may suggest the basic form of a strategy for 'teaching for transfer'.)

Another example is provided by a primary teacher aiming to harness the children's prior knowledge from outside school about counting objects etc., to lead to learning about addition in school mathematics (Walkerdine, 1988, Ch.6). Again, she shows how the process of 'translation' or 'transformation' of discourses must be accomplished through careful attention to the relating of signifiers and signifieds in particular chains of meaning. Thus,

> ... teachers manage in very subtle ways to move the children ... by a process in which the metonymic form of the statement remains the same while the relations on the metaphoric axis are successfully transformed, until the children are left with a written metonymic statement ....
> (Walkerdine, 1982, pp. 153-4)

However, transfer can go wrong if these issues are neglected, as illustrated by a primary school 'shopping game' (Walkerdine, 1988, Ch. 7). There a boy made 'errors' in his sums because he did not realise that, in the game, one was allowed - indeed, one was *required* by the rules, made to ensure the game's *pedagogic* effectiveness - to start afresh with a new 10p after each purchase. Though the child *called up* - that is, identified the task as - practical shopping, through which he 'made sense' of the apparent demands of the task, he nonetheless made errors because he was positioned in, and *regulated by*, the pedagogic shopping game.(NOTE 3)

While some aspects of everyday shopping practice were also useful in the game - say, remembering the familiar result that 'when you have 10p and buy something worth 9p, you will have 1p left', other aspects of shopping - for example, the knowledge of the requirement of giving up money to obtain a purchase - were not 'included' in the discourse of the school shopping game. Also, importantly, the goals and purposes were quite different in the two practices. Thus, Walkerdine (1988, pp. 115 ff) argues that activity within one discourse - say, playing the card game whist - will help with school maths in those, and only those, aspects of the game which are both contained in school maths and which enter into similar relations of signification (see also sec. 3.1).

3.3 Acknowledging the importance of affect, and the inter-relationship of thought, feeling and value

Most accounts of mathematical thinking, including those mentioned in Sec.2, largely ignore affect. There are a few exceptions. Taylor has discussed the power of desire at the individual and the societal level (e.g. 1989). Walkerdine has emphasised the importance of the relations between cognition and affect: 'meanings are not just intellectual' (Walkerdine & Girls and Maths Unit, 1989, p.52). Evans & Tsatsaroni (1996) have reviewed several different approaches to linking cognition and affect, and have argued for the approach presented here.

Affect can be seen as the energy behind much activity: 'Emotion powers reason' (Buxton, 1981). In this discussion, affect can be understood as an emotional charge attached to particular words, gestures, and so on. These signifiers are linked together to make up *chains of meaning.* Thus the charge can flow from one signifier to another, by *displacement*.

An example comes from an episode with an interviewee in my research (described below), 'Ellen'. When asked to calculate a 15% tip for a meal she has 'chosen' from a restaurant menu, she hesitates, then makes a 'slip' (dividing by 15, rather than multiplying). In response to a 'context-sensing question' about how often she does this sort of calculation, she admits that she doesn't usually pay, but nevertheless, she habitually adds up the cost of her meal - since she doesn't 'want to be an expense'. It is possible to read 'expense' as a signifier on which meanings are *condensed*: it would signify for Ellen both the cost of, say, her meal obtained by summing the individual dishes, and her being a burden within a relationship: these two ideas are *metaphorically* linked in her history. In this case the signifier 'expense' would be located at the intersection of two (at least) discourses (discourses on relationships, on eating out, and on school maths) The anxiety, guilt, pain of being an expense could be *displaced* onto the idea of the cost of her meal, and in turn onto any calculations entailed in producing that sum, including that of a tip. Here her displaced anxiety *may* influence her ability to transfer the calculation of 15% from school contexts (where she was a successful student) to everyday (eating out) or hybrid contexts (Evans and Tsatsaroni, 1994, 1996).

I would argue that the quality and intensity of affective charges may often be a major influence in the success or failure of many attempts at transfer - an influence that has so far been largely ignored in the mathematics education literature.

The inclusion of affect or emotion in the analysis of a particular subject's thinking is not just an 'optional extra'. Other work shows that whenever a teacher reaches outside of mathematics for an illustration, the mathematics is 'at risk'; e.g. when illustrating addition in the context of shopping with 'Mummy', if the mother 'has financial difficulties, ... is sick, far away or deceased' (Adda, 1986, p.59). This is because of the fundamental character of language, its ability to produce 'multiple meanings': the signifier can always break with any given context, and be inscribed in new contexts without limit (Evans and Tsatsaroni, 1994, p.184).

Thus another reason that a particular set of relations of signification may not provide fruitful inter-relations is that these relations may be distressing or distracting - and not only misleading.

Most of the examples of transfer discussed so far are examples of what is sometimes called *harnessing*, the use of knowledge *from out-of-school contexts in school*. Similar issues can be expected to arise in attempts to *transfer* from school learning to work or outside contexts; see the illustration and the case study below.

4. An Illustration: Looking for the Mathematics in Work

Noss & Hoyles (1996a) give a fascinating account of research done as preparation for teaching a course in 'banking maths' (BM) to employees of a major investment bank. They aim thereby to bring about changes in the work discourse by challenging it with academic maths - and also realise that this will challenge their position in academic maths (AM): '... to make sense of the world we were watching, we had to find a way to impose a mathematical gaze on it' (p.9).

In their general discussion of context, Noss and Hoyles emphasise mathematical representations (as do Nunes et al. - see below). They also problematise the characterisation of mathematics as involving straightforward decontextualisation and abstraction - thereby questioning further most traditional and constructivist positions - and focus on processes of 'situated abstraction' (1996b).

We can consider key points of their work in a way parallelling the discussion in the previous section:

(i) *a recognition that work practices and school maths discourses are distinct:* Noss and Hoyles found that the inhabitants of the bank 'spoke a different language....'(1996a, p.7). Further, in banking maths, *standards of accuracy* are distinctive (e.g. the tolerance of $25 allowed on large-scale transfers); certain *well-known results* can be used to avoid the need for calculations (e.g the fact that a Treasury Bill will have a lower purchase price than a simple interest 'instrument' at same rate of simple interest / discounting). Further, familiar representations, such as graphs, are 'read' differently: in BM, graphs tend to be considered as *displays of data* - whereas, in academic maths (AM), they are read as a 'medium for expressing relationships' (pp.13-15).

(ii) *locating relations of signification that provide fruitful points of inter-relation* in the learner's everday practices: One example is the care that is required in discussing interest calculations (using percentages), where the conceptual priority (in AM terms) of simple interest (prior to compound interest) conflicts with its relative rarity in BM practice (pp.12-13).

(iii) *identifying areas where work practices might usefully overlap with academic mathematics, and structuring the academic discourse so as to work systematically through the process of transfer*: Noss and Hoyles used the idea of a function as a 'bridging concept' between BM and AM, and used programming as a way of building models, so that their students would learn

> what it means to construct a mathematical relationship, and how and why the language of mathematics assists in conferring expressive power to the description of relationships.... programming is a way by which learners can express the state of their current understandings symbolically while holding on to the meanings which can all-too-easily become lost in the passage to conventional mathematical discourse.
>
> (Noss & Hoyles, 1996a, p.8)

Importantly, Noss and Hoyles also show how to develop new ideas on building bridges, by:
(iv) *developing a deeper mathematisation, by posing 'provocative' problems that appear 'innocent' in BM, but are deeply significant in AM:* For example, they use their knowledge of the meanings within the practical context (BM) to seize on the idea of 'continuous compounding', where the periods over which interest is calculated continually shrink (from yearly to monthly, to daily, etc.) and which is normally considered only as an 'exotic' topic in financial maths texts (1996a, p.20).

Thus Noss and Hoyles's work shows us how to apply and to enrich our developing notions of focussing on similarities and differences. They show how to enhance curriculum development with research, and to carefully construct the pedagogic discourse, so as to facilitate 'transfer'. Interestingly, this project combines harnessing knowledge from the practical context (for teaching purposes) with applying knowledge from the teaching situation to work practices.

5. A Closer Look at the Context

Nunes, Schliemann and Carraher (1993) have made great contributions to the study of context and of transfer. They have studied a wide range of occupations in their work settings - street-sellers, carpenters, farmers, fishermen, and so on - and pupils in the school setting. They have teased out several different levels of transfer, such as:

- application to problems with *unfamiliar* parameters, e.g. to non-standard ratios or scales on drawings for buildings (1993, Ch.5);
- *reversibility* or use of a procedure in the opposite direction from its usual use, e.g calculating a unit price, given the cost of *n* items; or
- *transfer across situation(s)*, e.g asking fishermen to solve unfamiliar ratio problems concerning the relationship between unprocessed and processed seafood that were isomorphic, in Nunes et al.'s view, to familiar problems about weight-price relationships (1993, pp.108ff.)

Nunes et al. have a range of theoretical antecedents (1993, Ch.7) including Gerard Vergnaud's (e.g. 1988) theory of concepts, in which structure ('invariants'), context ('situations'), and representation (especially oral vs. written modes) all play a role (see also Noss & Hoyles, 1996b, Ch.2). Their methodology has generally included both ethnographic description of the work practices of the relevant community, particularly the numerate aspects, and 'transfer experiments'. Here the subject is asked to solve problems that are familiar from their work, then to attempt others with requirements differing, for example, on one or more of the 'levels' of transfer indicated above. In one set of experiments Nunes et al. found that fishermen were able to use their everyday mathematics thinking in a conceptual, rather than just a procedural, way, to solve a range of problems, demonstrating (ii) reversibility and (iii) transfer (1993, pp.108-116).

Similarly, Schliemann and Carraher (1992) conclude that

> learners can develop proportional reasoning first in a limited range of contexts. ... Given the proper conditions, similarities of relations can be detected and transfer and generalisation become possible. This recognition may then act as a bridge for transfer of procedures to the unknown contexts.
>
> (Schliemann and Carraher, 1992, p. 61)

Schliemann (1995) concludes more generally

> ... mathematical knowledge developed in everyday contexts is flexible and general. Strategies developed to solve problems in a specific context can be applied to other contexts, *provided that the relations between the quantities in the target context are known by the subject as being related in the same manner as the quantities in the initial context are*. (p.49, my emphasis)

This is a similar conclusion to that reached by Walkerdine, on using experience with games like whist to help with learning maths (see above). But the argument in this paper points to analysing *both similarities and differences*.

In much of their reporting, Nunes et al. (e.g. 1993) have dealt with the complexity of the context of problem-solving by isolating out the 'situation' within which they assume the subject is thinking. Here the *situation* is understood as that part of the context that provides the overt background to problems - e.g as computation exercises, word problems or a 'store simulation'; the *context* additionally includes the setting and social relations - e.g. a clinical interview, or written test. The authors argue (e.g. 1993, Ch.3) that, while they are varying the situation across different problems, the context is held constant.

This separation of 'context' and 'situation' is an ingenious attempt to bring the crucial bases for thinking under the researcher's *control*: the experimenter allocates the *situation* for a given task in a controlled way, while the *context* of setting and social relations is assumed to be held constant over different situations. But this diminishes the importance of the context since the latter is kept away from the forefront of the analysis, and since its multiple facets (language, goals, social relations - see sec. 3.1) cannot be captured by the situation. The situation is considered to be given by the wording of the problem, and by any background information given by the interviewer, or taken-as-given by all subjects. Thus focussing on the situation in this way, at the expense of the context (at least in the transfer experiments) runs the risk of taking us back to traditional approaches (see Sec. 2).

Moreover, the context is not simply 'given' - the subject is involved in 'constructing', or construing, it! There may thus be significant variation in subjects' experience of the context. In particular, schooled subjects may *call up* school discourses as the basis for the context for their problem-solving. Nunes et al. acknowledge this possibility by recording the number of years of schooling for their subjects, and by comparing school students' and working-people's performances in some of their designs. But this may not adequately capture the likely variation across subjects in construing and experiencing the context. And this variation is not under the control of the researcher.

I aimed to take account of these problems in my research (Evans & Tsatsaroni, 1994; Evans, 2000), with college students, many of them over 21 years old, returning to study after some years of work or child-care. A set of semi-structured interviews focussed on maths “life-histories”, and on responses to several problems - e.g. reading graphs, deciding how much (if at all) they would tip after a restaurant meal, deciding which bottle of sauce would be the 'best buy' (cf. Sewell, 1981). These problems were chosen so that the context might be viewed either as mathematical, or as based on out-of-school practice(s).

I allowed that *the context of activity may be different for different subjects in the same setting*, by distinguishing the *practice(s) at play* in the setting from the *practice(s)* *called up* by different subjects. The *practice(s) at play* in the setting are considered to be the same for all subjects, whereas the *practice(s) called up* by different subjects may differ - even when they confront what appears to be 'the same task'.

It is the inter-relationship between the practice(s) at play and the practice(s) called up, a *positioning* based in language / discourse, that forms the context of a task for a particular subject. My analysis of my interview setting was that the two practices generally *at play* in the setting were: college maths, with subject-positions lecturer and student; and research interviewing, with subject-positions interviewer and respondent. Further, I considered that a particular student , in response to a particular interview question, might *call up* one of these two practices, or another practice e.g business discourses or eating out - or indeed, a combination of two or more practices. For further details, see Evans and Tsatsaroni (1994, 1996).

For this analysis, I needed to see whether sufficent information could be elicited from the interview to judge the context of students' thinking and affective responses to each of a range of problems. Here the use of *contexting* (or *context-sensing*) questions was a crucial innovation. When the student was first shown the 'props' for the problem - e.g. a facsimile of a graph from a newspaper in Question 3 (see Figure 1) - before being asked anything 'mathematical' - s/he was asked: 'Does this remind you of anything you currently do?'. And after discussing the question, s/he was asked 'Does this remind you of any earlier experiences?'. Subjects' responses to these questions helped me judge the context of their thinking about the problem, in the interview setting.

Figure 1 about here

To illustrate the method and some findings supportive of the argument here, I include an episode from one of the interviews. 'Donald' (a pseudonym) was male, in his 40s, with an O-level (age 16 qualification) equivalent in Maths. The background of his parents was working class; his own was middle class, having worked on the money markets in London.

In response to a graph showing how the price of gold varied over one day's trading (Figure 1), he begins by calling up business practices, or what might be called 'financial maths':

> JE : Does that remind you of anything that you do these days, or you've done recently?
> S : Er, some of the work we done in Phase One [the first semester of the College course], but if you ask me straight out of my head, what it reminds me of - I worked once with a credit company and we had charts on the wall, trying to galvanise each of us to do better than the other ....
> ....[8 lines omitted] ...
> That's what that reminds me of - a bad feeling in a way - I felt that a human being was being judged by that bit of paper (transcript, pp. 8-9)

Here we notice that Donald is reminded both of his 'College maths', and of his earlier managing of a sales team - but it is the latter business practices to which he gives priority in calling up. Note that calling up the business practices brings ('bad') feelings with it.

Next in the same episode, he mentions college maths, then seems to link 'financial maths' with it.

JE : ... Does it remind you of Phase One?
S : Yeah, well, we done some of the questions like this, and er, the run over the rise and that kind of thing...[5 sec.] ...trends, I suppose if you were judging a trend ... [2 lines omitted] ... I find good, I like the fact I can do a chart now ....

(transcript, pp. 10-11)

Here he uses the language of College maths, describing the gradient in terms of 'run' and 'rise'. He then shifts into work discourses, as evidenced by his use of the terms 'trend' (rather than 'gradient') and 'chart' (rather than 'graph'), which were not used in the college teaching.

Next I ask specific questions about the graph.

JE : Right, okay, may I ask you which part of the graph shows where the price was rising fastest?
S: If I was to make an instant decision, I'd say that one [i.e. before midday] - but obviously want to make it on a count of the line, wouldn't I?
JE: You'd?...
S: I'd count a line - as it goes up ... [25 sec.] ... eleven over six [for the increase before midday - see graph] and ten over six [for the increase before the close], so that one's right - in the first one ...
JE: ... [2 lines] And um, what was the lowest price that day?
S : This one here - five hundred and eighty ... went higher at the close, for some reason...

(transcript, p.12)

This reading shows several things relevant to my analysis above:

- Donald appears able and willing to use both college maths and financial maths; further he seems able to choose which practice to use to address the problem posed, to decide whether to apply his (more precise) college maths methods of calculating gradients to decide when the relevant price was rising faster.
- He is also aware of the *different goals* of the two practices, relating to different objectives in using the graph. In *business*, the objectives are implicitly competitive, to compare persons or groups - and growth-orientated, to make comparisons over time; in *college maths*, the aim is to analyse the qualities of the curve, including the rate of change. He is aware of *different values and standards of regulation*, in particular of *precision*, required in the two discourses.
- He is also open about the *different feelings* evoked by the two practices. For example, his awareness of the different goals of the two practices is sometimes painful.
- Donald is apparently able to focus on *discursive similarities and difference*s: he seems able to read the diagram as a 'chart' (business maths) or as a 'graph' (college maths), and to recognise the connections between a 'trend' and a 'gradient' (respectively). Though not certain, it appears that Donald is able to bridge the two practices, i.e. to transfer his college maths methods to help solve a problem involving charts - perhaps depending on some notion of *economy of cognitive effort* (cf. Pea, 1987).
- In this analysis, attention is drawn to the diagram, and to the role it might play in either discourse. Here it seems to provide a crucial representation, allowing one to focus on the similarities and difference between the discourses.

Thus, Noss and Hoyles's bankers, and Donald as a mature student, were able to appreciate the bridging between their learning in college (or academic) maths, and working practices, precisely because they were already familiar with the working practice towards which transfer was aimed. It is of course much easier to teach for applications, or to learn to undertake them, if one is clear about the contexts from which one is departing, and clear about those which one aims to reach.

6. Conclusion and Directions for Research

In this paper I take a sceptical, yet optimistic, position on the transfer of mathematical learning, as normally understood.

Continuity between practices (e.g. school and out-of-school activities) is not as straightforward as traditional views assume - and hence scepticism is in order about the claim that transfer is in principle straightforward. Responding to arguments from situated cognition and others, we can agree that there is a *distinction* - but not a total *disjunction* - between doing maths problems in school, and numerate problems in everyday life, and can acknowledge that transfer is not dependable. But it is not impossible; hence we can be more optimistic than these latter approaches suggest.

Although people do seem to transfer ideas, feelings, etc. from one context to another under all kinds of conditions - what they transfer is not always what we as educators would like them to transfer. Because of the vagaries of signification and also of emotional charges, anything like transfer will be difficult to predict or control - even with what seems the 'right' pedagogic or social support. *The ability of a signifier to form different signs, to take different meanings, within different practices, constitutes a severe limitation on the possibilities of transfer.*

Yet it also provides the basis for any such possibilities. Though the successful building of bridges cannot be guaranteed 'risk-free', this paper has sketched some steps it is *necessary* to follow. For anything like transfer to occur, a 'translation', a making of meaning, across discourses would have to be accomplished through careful attention to the relating of signifiers and signifieds, representations and other linguistic devices that are used in each discourse, so as to find those crucial ones that function differently - as well as those that function in the same way - in each. This translation is not straightforward, but it often will be possible.

Thus, bridges between practices can be built, by (a) describing the practices involved (in the transfer relationship), and analysing the related discourses, as systems of signs; and by (b) analysing the *similarities and differences* between discourses (e.g. school vs. everyday maths), so as to identify fruitful 'points of inter-relation' between school maths and outside ('target') activities.

In particular, unlike much previous work, the approach developed here emphasises the importance of affect, and inter-relationships of thought and feeling. The connections and the discontinuities between practices involve not only ideas, strategies, etc., but also values and feelings, carried by chains of signification. The cognitive and affective aspects of activity are formed and inter-related through the medium of language (NOTE 4).

Ways of designing pedagogic practices can be developed that will facilitate transfer, including: task analyses based on (b) above; incorporating a balance of generality and situational features in teaching the initial task; and providing practice on a range of initial and target tasks (NOTE 5).

To advance the study of transfer as reconceptualised, we need research programmes including a focus on sign systems and further studies of the workplace (cf. Nunes et al., 1993; Noss and Hoyles, 1996b) and of everyday activities (cf. Lave, 1988). My work contributes by showing how smaller-scale studies can also produce rich material spanning a wide range of practices. In particular, the *contexting questions* I used show how to reveal a wealth of associations between 'mathematical' problems and the subject's memories and constructions of experiences which provide a context for speaking of the meanings these problems have for him /her.

Mathematics and Statistics Group
Middlesex University
Enfield, Middx. EN3 4SF
U.K.

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Notes

1. Several of subjects in my interviews (see below) illustrate how failure to recognise this can hinder understanding of cognition. For example, for a 'best buy' question in the interview, a middle-class young man, calls up shopping practices - in which for him great value is not placed on saving a few pence, or indeed on saving money at all - and school maths is not called up.
2. It should be noted that a practice may be constituted by different discourses; for example, school mathematics may be constituted by transmission learning, or by problem-solving / investigative approaches. These different discourses will make available different versions of the teacher and the pupil positions. This point cannot be developed here, but, on different discourses of gender relations and corresponding subject-positions, see Hollway (1989).
3. Walkerdine does not suggest that the teacher's purpose in playing the shopping game was to produce transferable skills (from school to shopping), nor to 'harness' the children's (limited) experience with shopping for pedagogic purposes. But it was to give the children experience of action on money, or tokens, which could later be 'dis-embedded' in the process of producing abstract mathematical knowledge.
4. Fuller consideration of the examples here shows how psychoanalysis can allow insights into the affective (Walkerdine, 1988; Evans and Tsatsaroni, 1994, 1996; Evans, 1999a).
5. Methods of 'teaching for transfer cannot be further discussed here, but see Anderson et al. (1996), Masingila et al. (1996), and Evans (1999).

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***Figure 1*. Question 3 in the interview**

[3]

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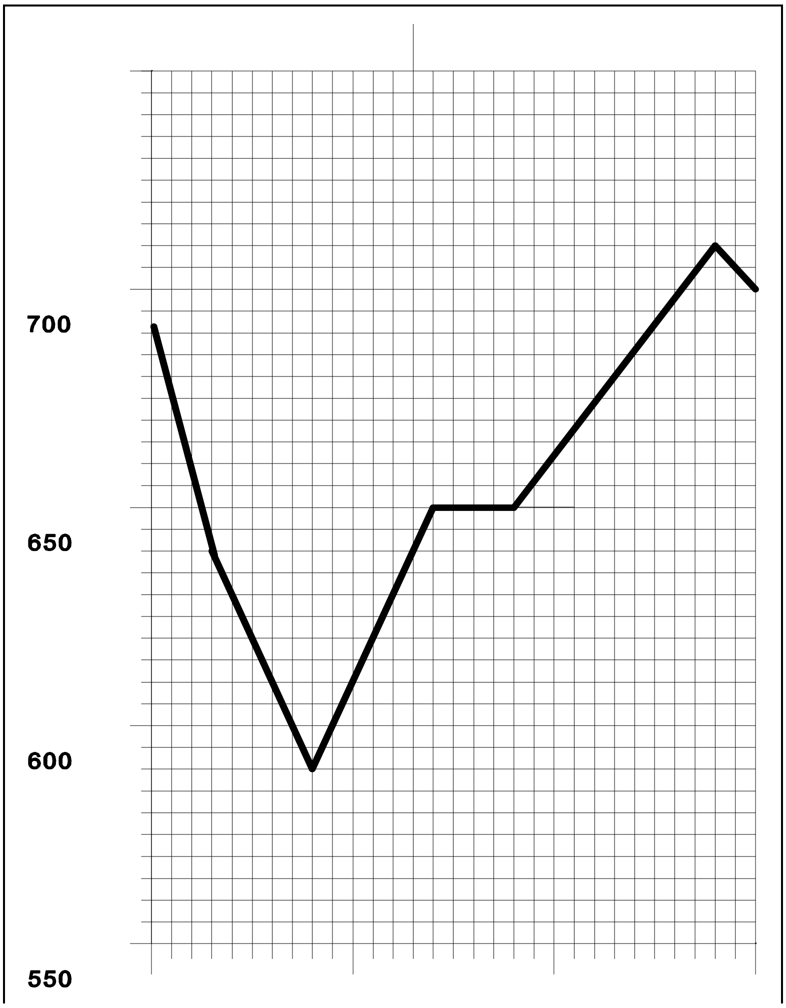
This graph shows how the price of gold (in dollars per
fine ounce) varied during one day's trading in London.
Which part of the graph shows where the price was rising
fastest? What was the lowest price that day?
```

The London Gold Price - January 23rd 1980



This graph shows how the price of gold (in dollars per fine ounce) varied during one day's trading in London.

***Source*: Evans (2000), based on Sewell (1981)**